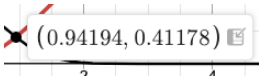
Period 3, March 7, 2025

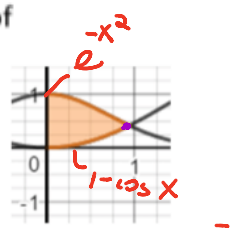


8. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis.

$x=0 \quad y=e^{-0^2}=e^0=1 \Rightarrow y=1-\cos 0=1-1=0$

a) Find the area of region R.

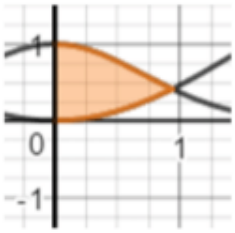
$\int_0^{.94194}\left(e^{-x^2}-(1-\cos x)\right)dx$

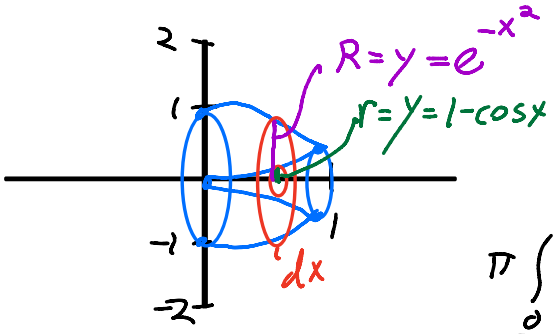


$\int_0^{.94194}\left(e^{-x^2}-(1-\cos x)\right)dx$

$= 0.590962450123$

b) Find the volume of the solid generated when R is revolved about the x-axis.





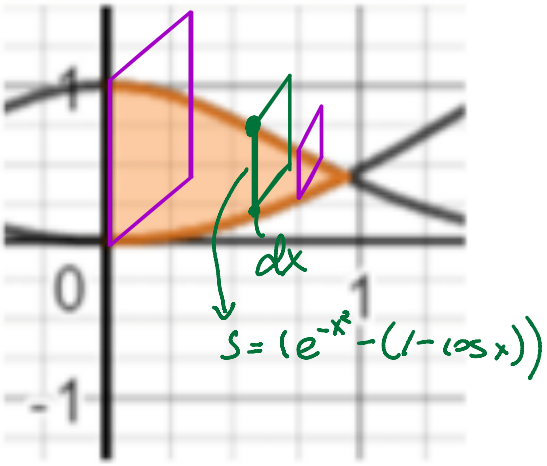
$\int_0^{.94194}\pi(R^2-r^2)\,dx$

$\pi\int_0^{0.94194}\left[\left(e^{-x^2}\right)^2-(1-\cos x)^2\right]dx$

$\pi\int_0^{.94194}\left(\left(e^{-x^2}\right)^2-(1-\cos x)^2\right)dx$

$= 1.74661409822$

c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



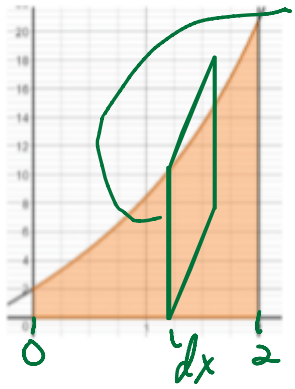
$\int_0^{0.94194} S^2\,dx = \int_0^{0.94194}(e^{-x^2}-(1-\cos x))^2\,dx$

$\int_0^{.94194}\left(\left(e^{-x^2}\right)-(1-\cos x)\right)^2 dx$

$= 0.461063510705$

7. Let R be the region enclosed by the x-axis, the y-axis, the line x = 2, and the curve $y = 2e^x + 3x$.

b) Find the volume of the solid whose base is R whose cross sections perpendicular to the x-axis are squares.



$y = 2e^x + 3x$

$$\int_0^2 (2e^x + 3x)^2 dx = \int_0^2 (4e^{2x} + 12xe^x + 9x^2) dx$$

$$\int_0^2 (2e^x + 3x)^2 dx$$

$= 231.864973253$

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9. {no calc} An object moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the object is at position $x = 2$ at time $t = 0$, what is the position of the object at time $t = 1$?

$\int v(T)\,dT = S(T)$

$\int (3T^2 + 6T)\,dT = T^3 + 3T^2 + C$ ($T = 0$, $x = 2$)

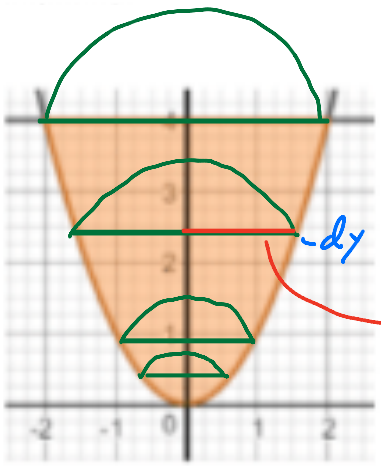
$2 = 0^3 + 3(0)^2 + C$

$2 = C$

$S(T) = T^3 + 3T^2 + 2$

$S(1) = 1^3 + 3(1)^2 + 2 = 6$

Find the volume of a solid that is bounded by the graphs of $y = x^2$ and y = 4 whose cross sections perpendicular to the y-axis are semicircles.



$\frac{\pi r^2}{2}$

dy

$r = ? = x$

$$\int_0^4 \frac{\pi r^2}{2}\,dy$$

$$\int_0^4 \frac{\pi \cdot x^2}{2}\,dy$$

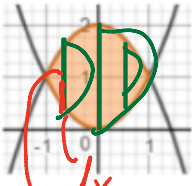
$$\frac{\pi}{2}\int_0^4 y\,dy$$

$$\frac{\pi}{2}\cdot\frac{1}{2}y^2\Big|_0^4$$

$$\frac{\pi}{4}(4)^2 - \frac{\pi}{4}(0)^2 =$$

$$\frac{\pi}{4}\cdot 16 - \frac{\pi}{4}(0) = \frac{16\pi}{4} = 4\pi$$

Find the volume of the solid that lies between the graphs of $y = x^2$ and $y = 2 - x^2$ whose cross sections perpendicular to the x-axis are semicircles.



dx

$= (2 - x^2) - (x^2)$

$$r = \frac{(2 - x^2 - x^2)}{2} = \frac{2 - 2x^2}{2} = \frac{2(1 - x^2)}{2} = 1 - x^2$$

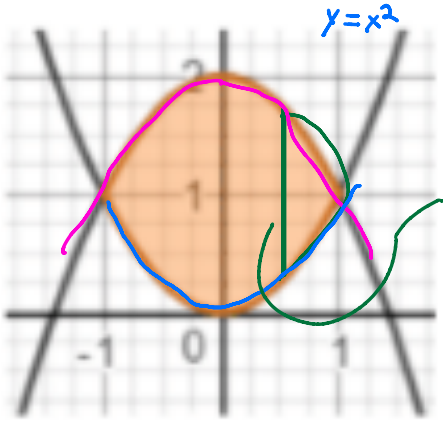
$$\int_{-1}^{1} \frac{\pi r^2}{2}\,dx = \int_{-1}^{1} \frac{\pi}{2}(1 - x^2)^2\,dx = \frac{\pi}{2}\int_{-1}^{1}(1 - 2x^2 + x^4)\,dx$$

$$\frac{\pi}{2}\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + C\Big|_{-1}^{1}\right)$$

$$\frac{\pi}{2}\cdot 2\left(x - \frac{2}{3}x^3 + \frac{1}{5}x^5\Big|_0^1\right)$$

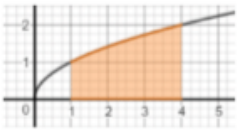
$$2\left(1 - \frac{2}{3}(1)^3 + \frac{1}{5}(1)^5\right)$$

$$2\left(\frac{15}{15} - \frac{10}{15} + \frac{3}{15}\right) = 2\cdot\frac{8}{15} = \frac{16}{15} = \left(\frac{16}{15}\right)\frac{\pi}{2}$$



3.

Find the volume of the solid that lies between the graphs of $y = \sqrt{x}$, $1 \le x \le 4$, and the x-axis whose cross sections perpendicular to the x-axis are isosceles right triangles where the base is the leg of a triangle.



$\frac{1}{2} \cdot b \cdot h$

$$\int_1^4 \frac{1}{2} \cdot y \cdot y \, dx = \int_1^4 \frac{1}{2} \cdot \sqrt{x} \cdot \sqrt{x} \, dx$$

$$\frac{1}{2} \int_1^4 x \, dx$$

$$\frac{1}{2} \cdot \frac{1}{2} x^2 \Big|_1^4$$

$$\frac{1}{4} x^2 \Big|_1^4$$

$$\frac{1}{4}(4)^2 - \frac{1}{4}(1)^2$$

$$\frac{16}{4} - \frac{1}{4} = \frac{15}{4} = 3\frac{3}{4}$$